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Experimental observation of coherence resonance in cascaded excitable systems

D. E. Postnov,[1] Seung Kee Han,[2] Tae Gyu Yim,[2] and O. V. Sosnovtseva[1,2]
[1]*Physics Department, Saratov State University, Astrakhanskaya Str. 83, Saratov 410026, Russia*
[2]*Department of Physics, Chungbuk National University, Cheongju, Chungbuk 360-763, Korea*
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Coherence resonance is investigated in a full-scale experiment using a monovibrator circuit. We present the effect of coherence behavior in an array of cascaded excitable systems. The regularity of the noise-induced oscillations is shown to increase along the chain. [S1063-651X(99)50904-5]

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The dynamics of noisy nonlinear systems displays a variety of complex phenomena. Among them are stochastic resonance ([1] and references therein) and stochastic ratchets ([2] and references therein) which are of great interest. Recently a noise-induced coherent motion has been observed for autonomous systems in [3], where the effect of noise on a nonuniform limit cycle has been studied, and in [4] where coherent resonance in an excitable system has been reported. Excitable dynamics is represented in different fields of applications ranging from chemical reactions to cardiac and nerve cells [5,6]. The excitable system has two characteristic times: the activation time τ_a and excursion time τ_e. It is typical for a wide variety of neurons whose bursting dynamics falls into distinct classes. The different sensitivity of those time scales to noise leads to their optimal relation, which causes the maximal regularity of output signal for a certain noise amplitude [4].

This paper provides an experimental observation of coherence resonance by means of an electronic monovibrator circuit. We show that two different types of monovibrators manifest themselves in different evolutions of the peak frequency in a power spectrum at a large noise amplitude. We emphasize that the cascaded systems demonstrate the gain of regularity along the chain.

To be specific we consider a monovibrator as an example of an excitable system that generates a single electric impulse when an excitatory signal exceeds the threshold level. Figure 1 represents two circuit implementations. The circuits employ (i) the operational amplifier, which realizes that a signum function is a nonlinear response to the voltage difference between two inputs and (ii) the capacitor C involved in the positive feedback, which provides a time-locking of the output circuit in an excited state via gradual voltage change in the ''+'' input. The characteristic time is the recharging time constant $\tau_0 = -RC\ln 0.5(V_b/E+1)$, where $V_b \leqslant E$ is the threshold voltage and E is the voltage of power supply. With switched on noise the monovibrator generates a single impulse of duration τ_0. Since weak noise is applied the system operates in a regime of random impulses of duration $\tau \leqslant \tau_0$. Figure 2 shows the time series of input noise and output signal. Note that strong enough noise can break down the process of impulse generation.

Applying Kirchhoff's rules [7] under some assumptions one can derive the model equations. The normalized equations for the (b) circuit [Fig. 1(b)] is as follows:

$$\dot{x} = \alpha y - \gamma x + D\xi(t),$$
$$\varepsilon \dot{y} = f(\dot{x} - v_b) - y, \tag{1}$$

where x and y represent the voltages at the ''+'' inputs and outputs of the operational amplifier, respectively, v_b is the normalized threshold voltage, α, γ are positive and are defined by the values of resistors R, R_1, R_2 [8], ε is a small positive parameter, and $f(\cdot)$ is a signum function, which takes values of $+1$ and -1 for the positive and negative arguments, respectively. The output voltage V_o used in experiment is proportional to $\dot{x}+x$.

To make the simplest bifurcational analysis of the system, let us replace the noise term in Eq. (1) by the constant value D and consider it a control parameter. In fact, it means the slow noise limit. The equations (1) have the single equilibrium point $P_0\{(D-\alpha)/\gamma, -1\}$. The linearization at this point gives the Jacobian matrix

$$\mathbf{J} = \begin{vmatrix} -\gamma & \alpha \\ 0 & -1/\varepsilon \end{vmatrix}, \tag{2}$$

with the eigenvalues $\lambda_1 = -\gamma$ and $\lambda_2 = -1/\varepsilon$, which are always negative. No bifurcations can occur when control parameters are varied. Thus, we exclude from consideration the case when applied noise can drive the system toward a bifurcation point and reveal a suprathreshold behavior.

It is interesting to see the influence of noise intensity on the features of averaged power spectra. For small noise intensity ($D \ll 100$ mV^2), a monovibrator generates the impulses of duration $\tau \approx \tau_0 = -RC\ln 0.5(V_b/E+1)$. The time

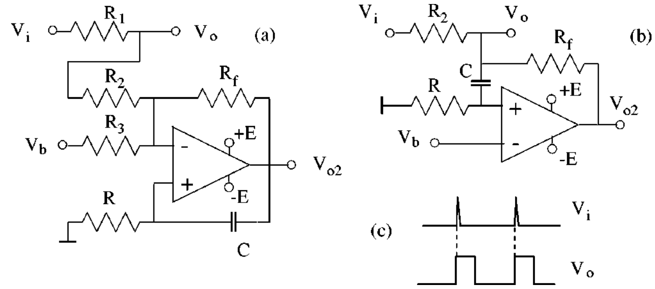

FIG. 1. (a) and (b) Two configurations of the electronic monovibrator circuit. (c) The output impulse is generated in V_o when V_i exceeds the threshold level.

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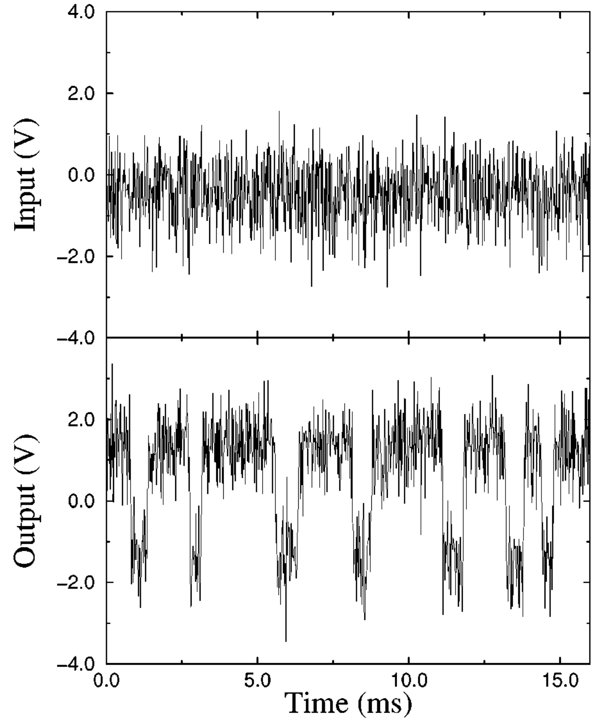

FIG. 2. Input and output signals for the monovibrator circuit.

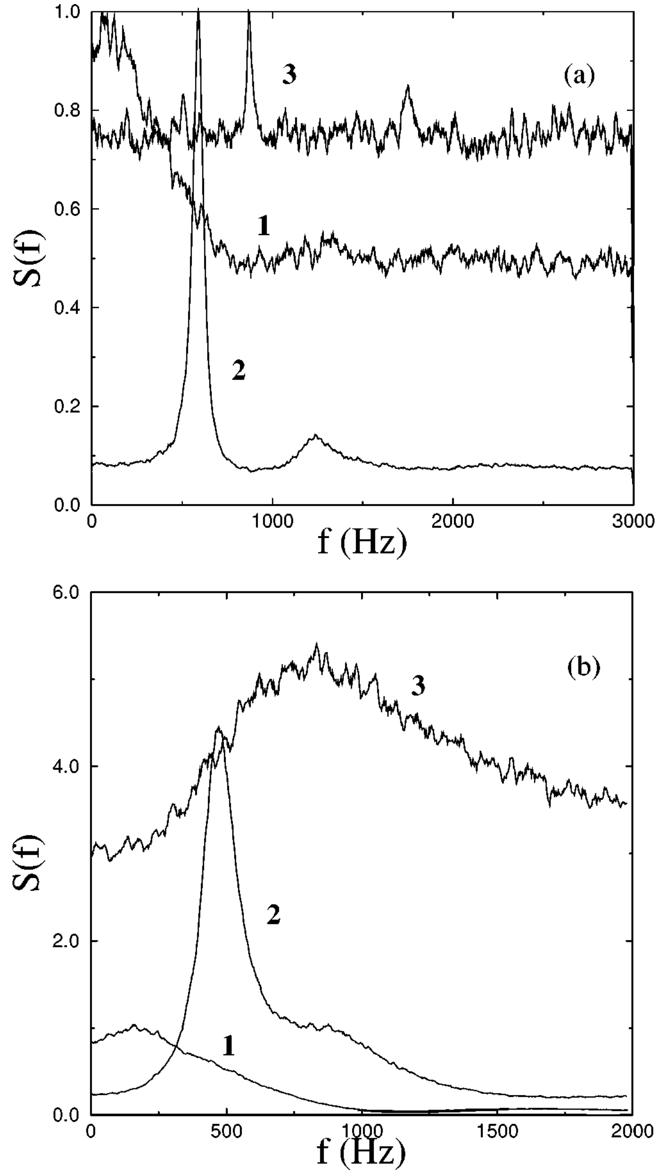

FIG. 3. Electronic experiment. The evolution of power spectra for the (a) circuit [Fig. 1(a)] and the (b) circuit [Fig. 1(b)] (curves **1**, **2**, and **3** correspond to $D=15$, 100, and 600 mV^2, respectively). For a more appropriate representation, the spectra (a) are normalized to their heights, while in (b) they keep their original values.

intervals between the impulses are much longer than τ. Thus, the resulting power spectrum is considered to be a superposition of impulses appearing randomly. The smooth and broad peak at low frequency can be observed in this case [Figs. 3(a) and 3(b), curves **1**].

For an optimal noise strength $D \approx 100\ \mathrm{mV}^2$ the pauses between impulses are approximately equal to their duration. The corresponding peak in the power spectrum is sharp and relatively high [Figs. 3(a) and 3(b), curves **2**]. Figure 4(a) illustrates how the peak frequency grows from 0 to $1.5/RC$ approximately as noise strength increases. Thus, we observe a noise-induced time scale of the system but not a noisy precursor of deterministic behavior.

To characterize the coherence behavior, we use a quantity that can be interpreted as the signal-to-noise ratio [3] $\beta = h/(\Delta\omega/\omega_p)^{-1}$, where h is the peak height normalized to the noise background at the frequency ω_p in the power spectrum, and $\Delta\omega/\omega_p$ corresponds to the relative width of the peak, which is the same as the familiar quality factor Q of a signal [9]. The β-D curve in Fig. 4(b) clearly shows a coherence resonance (CR) maximum as described by Pikovsky *et al.* [4]. It can be explained in terms of an optimal balance between the mean duration of an impulse generated by the monovibrator (excursion time τ_e) and the mean duration of a pause (activation time τ_a). The sensitivity of the system to noise within an impulse generation time is an essential parameter. This sensitivity in Eq. (1) is governed by the small parameter ε. In terms of electronic circuit, it is related to the changing of capacitor value C. The curves obtained for the three different C values are denoted as **1**, **2**, and **3** in Fig. 4. One can see that the increasing of C [decreasing of ε in Eqs. (1)] provides the gain of maximum regularity approximately according to the linear law $\beta_{max} \sim 1/\varepsilon$. Hence, CR is most pronounced in systems with strong relaxation properties.

For strong noise the pauses between impulse onsets tend to zero because the monovibrator is immediately pushed out from the equilibrium state. Moreover, the large noise can break the recharge process of capacitor C. Thus, the impulse duration becomes a random value. Finally, the additive component of noise manifests itself in power spectrum. This leads to a decrease in the regularity β when the noise intensity D increases above $100\ \mathrm{mV}^2$ [Fig. 4(b)].

We have to note that the evolution of a power spectrum for large noise levels depends on circuit configuration. For the (a) circuit (Fig. 1) the peak is absorbed by the increasing level of noise background but remains sharp [curve **3** in Fig. 3(a)]. In contrast, for the (b) circuit (Fig. 1) when the noise strength is increased, the peak width grows faster than its height, so that the peak becomes difficult to resolve from the noise background. [curve **3** in Fig. 3(b)]. What is the reason

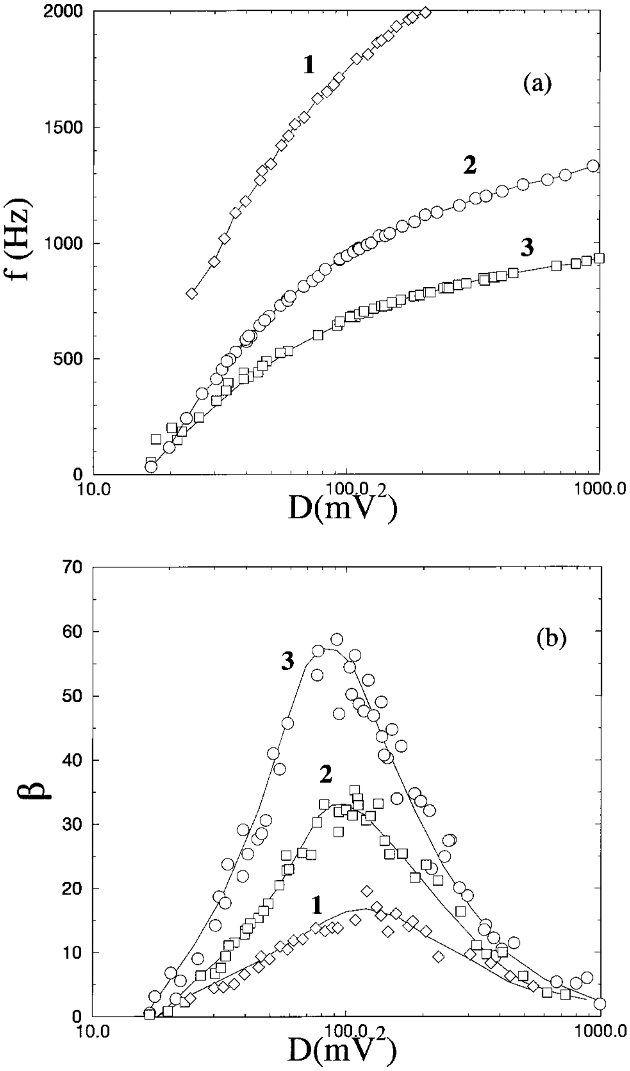

FIG. 4. Electronic experiment for the (a) circuit. (a) Peak frequency and (b) regularity β vs D for $C=0.01$, 0.02, and 0.03 μF (curves **1**, **2**, and **3**, respectively); $V_b=-1.016$ V.

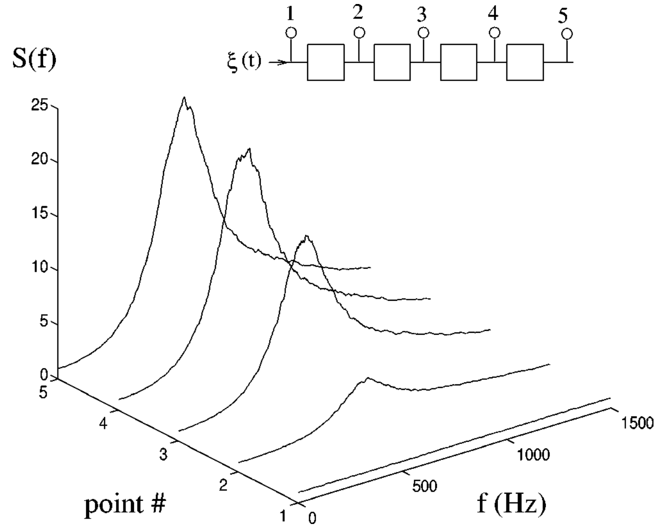

FIG. 5. Electronic experiment. The evolution of the power spectrum, which illustrates the regularity gain along the four-stage cascaded monovibrator circuit.

for such behavior from the circuit modeling point of view? The noise is presented in the output signal in two forms: (i) as an additive component that is responsible for the background level in the power spectrum and (ii) as a nonlinear response of a monovibrator. This means that for the case plotted in Fig. 3(a) the monovibrator generates impulses in a regular way even for the strong noise. In other words, the excursion time τ_e is almost independent of noise intensity. Examining the diagram for the (a) circuit one can make sure that the RC chain is separated from the noise input and the capacitor recharging rate is practically not affected by noise. In contrast, for the (b) circuit, the noise voltage, as well as the two-stage output voltage of the operational amplifier, is applied directly to the capacitor C. Thus, the recharging rate (i.e., the excursion time τ_e) strongly depends on the applied noise.

One of the most perspective fields for the application of the discussed CR effect is the stochastic neural network. A single neuron belongs to the class of excitable systems and the collective dynamics of coupled neurons governs the activity of living bodies. Therefore, we attempt to simulate the cooperative behavior in electronic experiment using the four-stage cascaded system. The first monovibrator is excited by the external noise, while each successive stage gets the input signal from the output of the preceding circuit. We calculate the power spectra from five sequential points of the cascaded system (inset in Fig. 5). It is clearly seen that the input signal (spectrum at point 1) looks like white noise. Figure 5 shows that the widespread peak in the power spectrum can be observed at the output point 2 of the first monovibrator. The relatively low value of regularity $\beta=5.05$ is related to the noise intensity (or, equivalently, the circuit parameters) as it is far from optimal value. Nevertheless, the following transmission of a signal through the cascaded monovibrators increases the regularity gain up to $\beta=22.49$ at point 5.

Thus, this phenomenon demonstrates that the cascaded systems, excited by white noise, are able to provide the spatial regularity gain of the desired level. It is known that for neural systems, the information about stimulus intensity is encoded in the frequency of generated impulses [10,11]. The observed effect of regularity gain can be considered one of the possible mechanisms for the creation of the information signal while the nerve impulses are transmitted along the cascaded neurons.

In summary, we have demonstrated that the nonlinear effect of a coherence resonance can be observed in an electronic experiment based on a monovibrator circuit. Two configurations of a monovibrator circuit provide different evolutions of a power spectrum when the noise intensity is increased. This is associated with relaxation properties of the system and which way a noise source is introduced. We have found the effect of regularity gain along the four-stage cascaded monovibrator circuits. The array of cascaded systems excited by δ-correlated white noise can form a regular output signal gradually along a spatial coordinate. We believe that array enhanced CR is an application for noise, which may be significant in biology and is important for engineers.

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[8] Kirchhoff's rules, applied to the (b) circuit (Fig. 1) with used values of electronic components, yield $\alpha=(1+R_2/R_1+R_2/R)^{-1}\approx 0.092$, $\gamma=(1/R_1+1/R_2)/(1/R_1+1/R_2+1/R)\approx 0.917$, $D=(1+R_1/R_2+R_1/R)^{-1}\approx 0.826$, and $\varepsilon=R'C'/RC\approx 2.33e-6$, where R' and C' are the output resistance and capacitance of the operation amplifier chip. The normalized time t/RC is used in Eqs. (1).

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